# Convolutional Sparse Coding Network via Improved Proximal Gradient for Compressed Sensing Magnetic Resonance Imaging


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## Introduction

Magnetic resonance imaging (MRI) may introduce artifacts in the imaging process due to the long data acquisition time and slow imaging speed in the k-space, resulting in a decline in imaging quality.

CSMRI is an ideal solution for MRI reconstruction. Still, the common method of sparse representation of image blocks has some defects: it ignores the correlation between image blocks, resulting in high redundancy in the final results and losing some important details and textures of MR images.

We choose the slice based convolutional sparse coding (CSC) model to compensate for the shortage of sparse coding by establishing translation invariance.

## Methods

- The variance regularization term is introduced into the CSC problem to remove the constraint on the convolutional dictionary.
- Introduce the heavy ball system with dry friction from the dynamic system perspective to find a better local optimal solution.
- The improved proximal gradient algorithm is unfolded into an encoder network to obtain the coding. The convolutional dictionary is updated by the backpropagation algorithm via the mean square error of the reconstructed signal.

## Methods(continued)

The improved optimization problem:

$$\arg\min_{\{\boldsymbol{\alpha}_{l,i}\}} \frac{1}{2}\sum_{l=1}^{I}\left\|\mathbf{F}_u\sum_{i=1}^{N}\mathbf{P}_i^T\mathbf{D}_L\boldsymbol{\alpha}_{l,i}-\boldsymbol{y}_l\right\|_2^2 + r_1\sum_{l=1}^{I}\sum_{i=1}^{N}\|\boldsymbol{\alpha}_{l,i}\|_1 + r_2\sum_{i=1}^{N}\left[\left(T-\sqrt{\mathrm{Var}(\boldsymbol{\alpha}_{\cdot i})}\right)_+\right]^2$$

*For this $f+g$ type composite optimization problem:*

$$\nabla_{\boldsymbol{\alpha}_{a,b}} f = \begin{cases} \mathbf{D}_L^T\mathbf{P}_a\mathbf{F}_u{}^T\left(\mathbf{F}_u\sum_{i=1}^{N}\mathbf{P}_b^T\mathbf{D}_L\boldsymbol{\alpha}_{a,i}-\boldsymbol{y}_a\right)-\frac{2\beta}{I-1}\frac{T-\sqrt{\mathrm{Var}(\boldsymbol{\alpha}_{\cdot b})}}{\sqrt{\mathrm{Var}(\boldsymbol{\alpha}_{\cdot b})}}(\boldsymbol{\alpha}_{a,b}-\boldsymbol{\mu}_b), & \sqrt{\mathrm{Var}(\boldsymbol{\alpha}_{\cdot b})}<T \\ \mathbf{D}_L^T\mathbf{P}_a\mathbf{F}_u{}^T\left(\mathbf{F}_u\sum_{i=1}^{N}\mathbf{P}_b^T\mathbf{D}_L\boldsymbol{\alpha}_{a,i}-\boldsymbol{y}_a\right), & otherwise \end{cases}$$

$$T_a(x) = \begin{cases} x-\lambda(r_1+r_3) & x>\lambda(r_1+r_3)+a \\ a & \lambda(r_1-r_3)+a<x\le\lambda(r_1+r_3)+a \\ x-\lambda(r_1-r_3) & \lambda(r_1-r_3)<x\le\lambda(r_1-r_3)+a \\ 0 & -\lambda(r_1+r_3)<x\le\lambda(r_1-r_3) \\ x+\lambda(r_1+r_3) & x\le-\lambda(r_1+r_3) \end{cases}$$

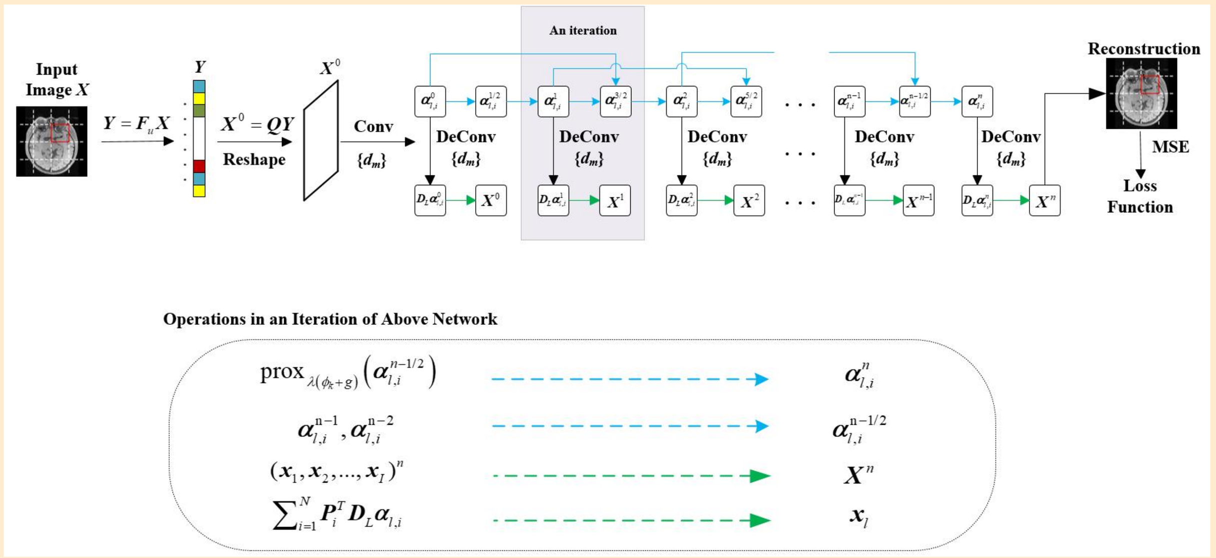
*$T_a(x)$ is one-dimensional decomposition of $\mathrm{prox}_{\lambda(\phi_k+g)}(x)$.*

Iterative steps:

$$x^{k+1} = \mathrm{prox}_{\lambda(\phi_k+g)}\left(x^{k+1/2}\right)$$

$$x^{k+1/2} = x^k + \beta\left(x^k - x^{k-1}\right) - \lambda\nabla f\left(x^k\right)$$

*where $\lambda = \frac{h^2}{1+\gamma h}$, $\beta = \frac{1}{1+\gamma h}$.*

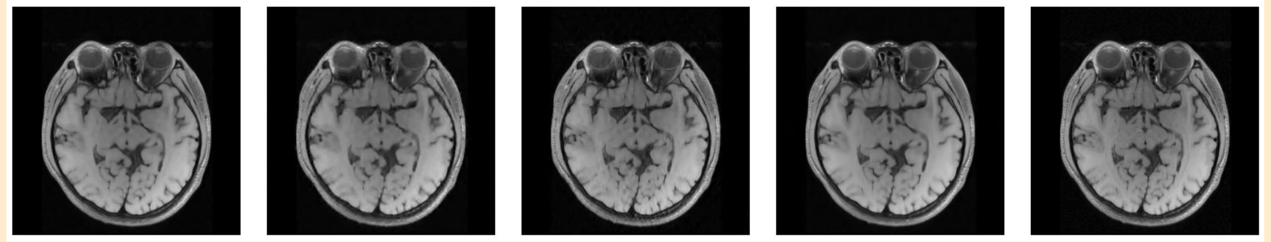



*The framework of the proposed iterative neural network design via unfolding CSC.*

## Results

Reconstruction performance:

| Methods | Undersampling ratios | | | | |
|---|---|---|---|---|---|
| | 10% | 20% | 30% | 40% | 50% |
| Proposed | **38.46** | **39.75** | **41.32** | 42.47 | 43.79 |
| ADMM-Net | 26.98 | 29.74 | 31.82 | 34.23 | 35.32 |
| $l_0$-CSC-Net | 27.70 | 31.92 | 34.09 | 35.55 | 36.89 |
| ISTA-Net+ | 37.16 | 38.73 | 40.89 | **42.52** | **44.09** |



*Reconstruction results of the image under different noise levels. From left to right are of ISTA-Net+, ADMM-Net, l0-CSC-Net, IDPCNN and our method.*

Average reconstruction time:

| Methods | Proposed | ADMM-Net | $l_0$-CSC-Net | ISTA-Net+ |
|---|---|---|---|---|
| Times(seconds) | 0.0618 | 0.8026 | 0.0698 | 0.1467 |

## Discussion

- In most cases, our method performs better than other methods. This may be because, for higher sparsity, using variance regularization for code can better use the convolutional network's multi-layer structure.
- The reconstruction time data show that our method can complete reconstruction at a fairly fast speed.

## Conclusion

The proposed model-based structured deep network takes advantage of CSC and deep learning. The improved ISTA-IDF algorithm is introduced into the CSC model and extended to a depth network, improving the MRI reconstruction process's convergence rate.

Compared with the existing methods, the proposed network performs better at low sampling rates, ensures clearer reconstructed images, has more detailed information, and speeds up the reconstruction process.

## References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