Supplementary Material for Optimal Camera Configuration for Large-Scale Motion Capture System

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1 Supplementary Material

Proof of Theorem 1 Inspired by [1], let $\mathfrak{G}$ consist of $k \cdot k!$ points. For the set $\mathfrak{C}$ of all subsets $\mathfrak{C}(\mathfrak{i})$ will have $k!$ sets, which contains the optimal subcover $\mathfrak{C}^*$ and $k! \sum_{j=1}^{k}(\frac{1}{j})$ additional sets making up a subcover $\mathfrak{C}'$ that our proposal might choose.

Divide $\mathfrak{G}$ into k segments of $k!$ points each labeled from 1 to k, if there exist a maximum overlap $S, |S| > 1$, the optimal subcover $\mathfrak{C}^*$ will consist of $k!$ overlapping sets, each containing one point from each of the k segments.

The choosable subcover $\mathfrak{C}'$ is made up of $k!/k$ overlapping k-element sets forming a cover of segment k, $k!/(k-1)$ overlapping $(k-1)$-element sets forming a cover of segment $k-1$, ..., and $k!$ overlapping single element sets forming a cover of segment 1 [1].

Our proposal could choose the $k!/k$ sets covering segment k first, since each will cover k points from other subsets. After these have been chosen, no remaining set covers more than $k-1$ new points, so our proposal could next choose the $k!/(k-1)$ sets covering segment $k-1$. This process could continue until our proposal had chosen the entire subcover $\mathfrak{C}'$ with $k! \cdot (1/k + 1/(k-1) + ... + 1)$ sets, and so the lower $\sum_{j=1}^{k}(\frac{1}{j}) \leq H_k$ follows.

For the upper bound, $\mathfrak{C}$ is the set of any combination of the $\mathfrak{C}(i)$, all of whose sets have no more than k elements, for camera configurations, $\mathfrak{C}$ corresponds to any group of camera configurations. Let $\mathfrak{C}^*$ be the optimal camera configuration. Since $\bigcup_{\mathfrak{g} \in \mathfrak{C}(i)} \mathfrak{g} = \mathfrak{G}$, we have $\mathcal{O}(\mathfrak{C}^*) = a|\mathfrak{G}|$, $\exists a, a \in [|\mathfrak{G}|/k, k], |\mathfrak{C}^*| <= M, |\mathfrak{C}(i)| <= k, k \in \mathbb{R}^+$.

If $\mathfrak{C}'$ is a configuration generated by our proposal on $\mathfrak{G}$, and let us analyze the complexity of the algorithm in a particular process during which $\mathfrak{C}'$ is chosen. For each grid point $\mathfrak{g} \in \mathfrak{C}'$, let the overlapping set $m(S)$ be the value of $\left|\bigcup_{j=1}^{M}((S \cap \mathfrak{C}(j))\right|$ when S was added to P by our proposal, and we have

$$m(S) = \left|\bigcup_{j=1}^{M}(S \cap \mathfrak{C}(j))\right| = H_k \cdot |\mathfrak{G}|.$$

Let us consider the performance of our proposal on $\mathfrak{G}$ and explore some of its specific relationships with the optimal configuration $\mathfrak{C}^*$ in more depth. If at a given time a set S' is chosen with ratio $\kappa(S') = |\bigcup_{j=1}^{M}((S' \cap \mathfrak{C}(j))|/|S' \cap \mathfrak{C}^*| \geq y_0$, then at this time each $S' \in \mathfrak{C}^*$ can have at most $1/(y_0+1)$ of its point in $\mathfrak{G} \setminus \mathfrak{C}(i)$. For if $S' \in \mathfrak{C}^* \cap P$, S has no points in $\mathfrak{G} \setminus \mathfrak{C}(i)$; otherwise we must have $\kappa(S') \geq y_0$, so $|S' \cap (\mathfrak{G} \setminus \mathfrak{C}(i))|/|S'| \leq 1/(y_0+1)$. Thus,

$$|\mathfrak{G} \setminus \mathfrak{C}(i)| \leq \left| \bigcup_{S' \in \mathfrak{C}^*} S' \cap \mathfrak{G} \setminus \mathfrak{C}(i) \right| \leq \frac{\mathfrak{C}^*}{y_0+1} = \frac{a \cdot |\mathfrak{G}|}{y_0+1}.$$

And therefore, more than $|\mathfrak{G}|(1-a/(y_0+1))$ grid points that need to be covered are not in the set $\mathfrak{G} \setminus \mathfrak{C}(i)$. Thus, we cannot choose an S' with the ratio $\kappa(S') \geq y_0$ until $1-a/(y_0+1)$ grid points have been covered. Conversely, if the ratio $\kappa' = |\mathfrak{C}(i)|/|\mathfrak{G}|$, the ratio κ for the next set $\mathfrak{C}_{k'}$ chosen will not greater than $a/(1-\kappa')-1$.

The ratio $\kappa(S)$ establishes a mathematical model of how the overlap can exist between newly added points and already covered points, and how to maximize the overlapping area where the common points have for $\mathfrak{C}(i)$.

We quantify the complexity of computing the maximum overlap point by point. The size of overlap contributed by the corresponding point is measured by the ratio $\kappa(S)$. If $\kappa(S) \geq y_0$, the cumulative overlap $|\mathfrak{C}(i)| \leq a-1$, the percentage of grid points that need to be covered are not in the set $\mathfrak{G} \setminus \mathfrak{C}(i)$ will be $h'(t) = 1-a/(t+1)$. Otherwise, for $\kappa(S) \leq y_0$, where $t \in [a-1, T']$, $h'(t) = 0$. The contribution function from each point is a step function $h'(y)$ on the interval $[0, T'], T' = |\mathfrak{G}|$.

$$h'(t) = \begin{cases} 1 - \frac{a}{t+1}, t \in [0, a-1], \\ 0, t \in [a-1, T']. \end{cases}$$

t is considered as the serial number of the point that was chosen from the ratio $\kappa(S) \geq y_0$, and we have

$$\begin{aligned} m(S) &= \frac{|\mathfrak{G}|}{a} \cdot \int_0^{a-1} h'(t)dt = \frac{|\mathfrak{G}|}{a} \cdot \int_0^{a-1} (1 - \frac{a}{t+1})dt = \frac{|\mathfrak{G}|}{a} \cdot [t - a\ln(t+1)]_0^{a-1} \\ &= |\mathfrak{G}| \cdot (1 - \frac{1}{a} - \ln a) \leq |\mathfrak{G}| \cdot \left[(\ln k + 1) - \ln|\mathfrak{G}| - \frac{1}{a} \right] \leq |\mathfrak{G}| \cdot (\ln k + 1). \end{aligned}$$

From the property of harmonic series ,

$$(\ln k + 1) \leq \left[\sum_{j=1}^{k} (\frac{1}{j}) + \frac{1}{2} \right].$$

Thus, the upper bound follows.

References

[1] David S Johnson. Approximation algorithms for combinatorial problems. *Journal of computer and system sciences*, 9(3):256–278, 1974.